



GCE AS/A Level

0975/01



S17-0975-01

MATHEMATICS – C3
Pure Mathematics

WEDNESDAY, 7 JUNE 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_5^7 \ln(1+x^2) dx.$$

Show your working and give your answer correct to one decimal place.

[4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_5^7 \ln\left(\frac{3}{\sqrt{1+x^2}}\right) dx.$$

[3]

2. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$6 \tan^2 \theta - 6 = 4 \sec^2 \theta + 5 \sec \theta.$$

[6]

- (b) Find all values of ϕ in the range $0^\circ \leq \phi \leq 360^\circ$ satisfying

$$3 \sec \phi + 5 \tan \phi = 0.$$

[3]

3. (a) Given that

$$x^4 - 3x^2y + 2y^3 - 4x = 7,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y .

[4]

- (b) Given that $x = 7t + 2t^2$, $y = \frac{4+3t}{7+4t}$,

(i) show that $\frac{dy}{dx} = \frac{k}{(7+4t)^n},$

where the values of the constants k and n are to be found,

(ii) find a similar expression for $\frac{d^2y}{dx^2}.$

[8]

4. A large tank in the form of a cuboid is used to store water. The width of the tank is denoted by x m. The length of the tank is 4 m **greater** than its width, whilst the height of the tank is 2 m **less** than its width. The volume of the tank is 150 m^3 .

(a) (i) Show that $x^3 + 2x^2 - 8x - 150 = 0$.

(ii) Show that $5 < x < 6$.

[4]

- (b) The recurrence relation

$$x_{n+1} = (150 + 8x_n - 2x_n^2)^{\frac{1}{3}},$$

with $x_0 = 6$, can be used to find the value of x . Find and record the values of x_1, x_2, x_3, x_4 . Write down the value of x_4 correct to two decimal places and prove that this is the value of x correct to two decimal places.

[5]

5. (a) Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(i) $\sqrt{3x^2 + 5x}$

(ii) $\sin^{-1} 3x$

[4]

- (b) By first writing $y = \cot^{-1} x$ as $x = \cot y$ and then assuming the derivative of $\cot y$, find $\frac{dy}{dx}$ in terms of x .

[4]

6. (a) Find each of the following integrals, simplifying your answer wherever possible.

(i) $\int 8e^{2-5x} dx$

(ii) $\int \frac{6}{\sqrt[3]{4x-7}} dx$

(iii) $\int \cos\left(\frac{7x-9}{3}\right) dx$

[6]

- (b) (i) Differentiate $\ln(3x^2 - 8)$ with respect to x .

- (ii) Use your answer to (b)(i) to evaluate

$$\int_2^6 \frac{3x}{3x^2 - 8} dx.$$

Give your answer in the form $\ln k$, where k is an integer whose value is to be found.

[6]

TURN OVER

7. (a) Show, by counter-example, that the following statement is false.

$$\text{'If } \frac{7x-200}{x} > 5, \text{ then } x > 100.\text{'}$$
 [2]

- (b) The graph of $y = f(x)$ has a single maximum which is situated at the point $(-2, 4)$. The graph of $y = af(x + b)$ has a single minimum which is situated at the point $(4, -2)$. Find the values of the constants a and b . [2]

8. The function f has domain $[8, \infty)$ and is defined by

$$f(x) = 2 + \frac{3}{\sqrt{5x-4}}.$$

- (a) Find an expression for $f^{-1}(x)$. [4]
 (b) Write down the domain of f^{-1} . [2]

9. The function f has domain $[2, \infty)$ and is defined by

$$f(x) = 4x + k,$$

where k is a constant.

- (a) Write down, in terms of k , the range of f . [1]

The function g has domain $[-3, \infty)$ and is defined by

$$g(x) = x^2 - 9.$$

- (b) Find the least value of k so that the function gf can be formed. [2]

- (c) (i) Write down an expression, in terms of k , for $gf(x)$.

- (ii) Given that $gf(2) = 7$, find the value of k . [5]

END OF PAPER